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# MATHEMATICAL MODELING OF HEAT TRANSFER PROCESSES AT LASER HARDENING BASED ON METHODS OF POLYARUGUMENT SYSTEMS

***The results of computer simulation of three-dimensional temperature fields of the cutting tool under the conditions of its hardening by continuous laser radiation are presented. The main provisions of the construction of the polyargument systems method used in solving the considered quasi-stationary heat transfer problem are given. The results of computational experiments to study the effects on the thermal state of the cutting tool of the distance a* between the cutting edge and the center of the laser beam are analyzed. According to the data of the conducted studies, it has been shown that a change in the value of a* has a significant effect on the pattern of the temperature fields in the tool being treated. The results of calculated studies and nature experiments concerning the dependence of the dimensions of the hardened zone on the distance a* between the cutting edge and the center of the heating spot are presented. The physical interpretation of the data obtained based on the analysis of the action of a number of competing factors are given.*** [dx.doi.org/10.29010/086.3]

Keywords: *laser hardening; heat transfer processes; methods of polyargument systems; mathematical modeling; cutting tool.*

## Introduction

Laser hardening of materials, as you know, is one of the main laser applications in mechanical engineering. The widespread use of laser hardening is associated mainly with two circumstances, firstly, with a number of advantages of this hardening method, such as the possibility of selective hardening, the presence of relatively small deformations of tools after heat treatment, the possibility of sufficiently precise control of the depth of the hardened layer, etc. and, secondly, with the fact that for surface hardening lasers with an average power of 1–5 kW can be used, which are relatively simple both in production and in operation [1–4].

The widespread use of laser hardening has led to the need for detailed theoretical and experimental studies of different processes occurring in the material when exposed to laser radiation. Among these studies an important place is occupied by thermophysical researches, since it is precisely the nature of thermal processes that largely determines the possibility of achieving the desired technological result.

## Formulation of research objectives

The purpose of the article is to establish the patterns of heat transfer processes flow during laser hardening of the cutting tool.

## Task setting and research methodology

The object under study (cutting tool) is a wedge-shaped body, fig. 1.

The surface normally distributed source of laser heating moves with a constant velocity $V$ parallel to the $z$ axis at some distance from it. The heat transfer conditions of the cutting tool with the environment are described by Newton-Richman law. In this case, the ambient temperature $t_a$ and the heat transfer coefficient $\alpha$ are assumed to be constant and the same for all limiting surfaces.

According to the above, the mathematical model of the heat transfer process under study during the hardening of the cutting tool with a laser beam for ultimate quasi-stationary state can be represented as.

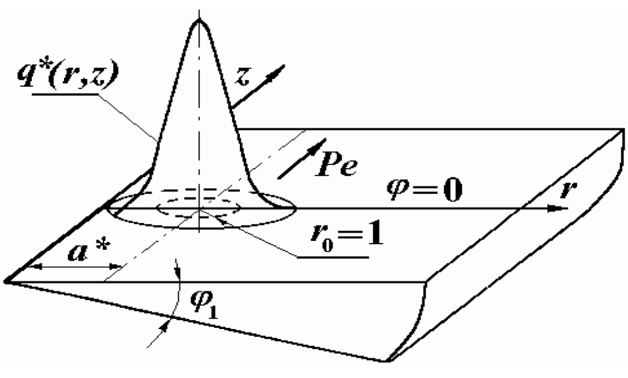


**Fig. 1.** To the formulation of the problem

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\Lambda\frac{\partial\theta}{\partial r}\right)+\frac{1}{r^2}\cdot\frac{\partial}{\partial\varphi}\left(\Lambda\frac{\partial\theta}{\partial\varphi}\right)+ +\frac{\partial}{\partial z}\left(\Lambda\frac{\partial\theta}{\partial z}\right)+Pe\cdot C_V\frac{\partial\theta}{\partial z}=0, \tag{1}$$

$$\Lambda\frac{\partial\theta}{\partial r}-Bi\cdot\theta\big|_{r=r_e}=0, \tag{2}$$

$$\Lambda\frac{\partial\theta}{\partial r}+Bi\cdot\theta\big|_{r=r_i}=0, \tag{3}$$

$$\Lambda\frac{1}{r}\cdot\frac{\partial\theta}{\partial\varphi}-Bi\cdot\theta\big|_{\varphi=0}=-\exp\left[-z^2-\left(r-a^*\right)^2\right], \tag{4}$$

$$\Lambda\frac{1}{r}\cdot\frac{\partial\theta}{\partial\varphi}+Bi\cdot\theta\big|_{\varphi=\varphi_1}=0, \tag{5}$$

$$\Lambda\frac{\partial\theta}{\partial z}\bigg|_{z=\pm\infty}=0, \tag{6}$$

here $\theta=(t-t_a)\lambda_0/(q_{\max}\cdot r_{0d})$; $Pe=V\cdot r_{0d}/a_0$; $Bi=\alpha\cdot r_{0d}/\lambda_0$; $r=rd/r_{0d}$; $r_e=r_{de}/r_{0d}$; $r_i=r_{di}/r_{0d}$; $a^*=a^*_d/r_{0d}$; $Z=z_d/z_0$; $\Lambda=\lambda/\lambda_0$; $C_V=c_V/c_{V_0}$; $a_0=\lambda_0/c_{V_0}$; $q_{\max}$ — is the maximum value of the heat supply diagram; $t_a$ is the ambient temperature; $V$ is the velocity of the laser beam; $r_{0d}$ is the radius of the heating spot; $r_{di}$, $r_{de}$ — internal and external radius of the detail; $a^*_d$ — the distance from the cutting edge to the location of the maximum heat supply; $\varphi_1$ — angle sharpening tool; $\lambda_0$, $c_{V_0}$ — fixed values of thermal conductivity and specific volumetric heat capacity of the material, respectively.

An integral polyargument systems method (MPS), focused on solving multidimensional heat transfer problems was used to solve indicated problem [5—6]. This method is based on three main provisions:

1. Elimination of the need to use any a priori elements in the sought-for solution and determining to the maximum extent possible all the information required for building the solution, based only on the given mathematical formulation of the problem.
2. Realization of the completeness of the functional reflection of the initial information in the reduced task.
3. Reduction of a multidimensional problem to special one-dimensional problems.

The first provision deals with the exclusion of such a priori elements as basic functions, as well as some additional functions that ensure satisfaction of boundary conditions, taking into account the shape of the region, etc. As for the second provision, it represents the requirement that the information appearing in the original mathematical model in a functional form, was reflected in the reduced formulation also in a functional form. The third position is motivated by the need to have a well-developed mathematical apparatus that can be effectively applied to the solution of the reduced problem.

## Summary of the main research results

The task of choosing the rational technological modes of laser hardening of the cutting tool is associated with the organization of certain temperature regimes of the hardened detail, which ensures the efficient flow of the required structural changes in the material. In this regard, it is of interest to analyze the possibility of using various methods of influencing the temperature regime of the cutting tool in the process of its hardening. Such methods include, in particular, the variation of the distance $a^*$ between the cutting edge and the center of the heating spot.

Below are the results of numerical studies relating to – the analysis of the effects of the magnitude of $a^*$ on the thermal state of the cutting tool.

Typical data of performed computational experiments are presented in Fig. 2—3. The given results correspond to the following values of the initial parameters: Bi = $1.064\cdot10^{-3}$; $r_e$ = 100; $r_i$ = 0.001; $\varphi_1=\pi/3$; $r_0$ = 1.0; Pe = 4.5; material of the cutting tool — steel U8. (Here, the quantity $r_{0d}$ ($r_{0d}=2\cdot10^{-3}$m) was taken as the characteristic size in the Bi complex, the simplexes $r_e$, $r_i$ and $a^*$). The value of $a^*$ varied in the range of 0 ... 1.5.

When comparing situations corresponding to different values of $a^*$, the values of the maximum density of the input flow $q_{\max}$ and the radius of the heating spot $r_0$ were assumed to be the same. Obviously, under these conditions, some of the laser radiation that does not fall

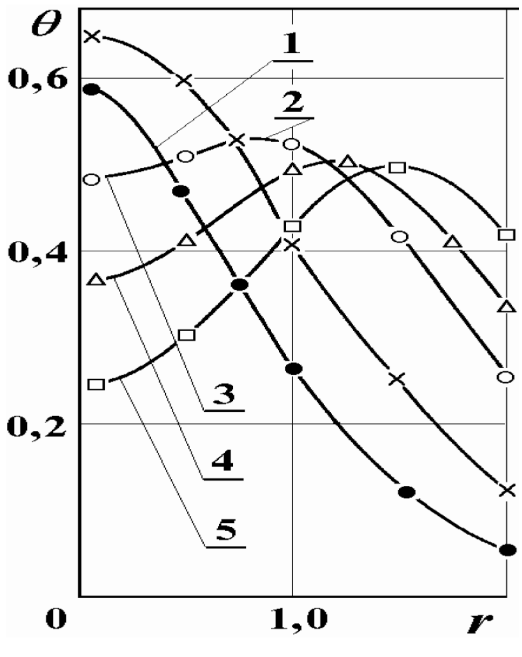


**Fig. 2.** The temperature change of the cutting tool in the radial direction when $\varphi=0$; $z=-0.5$; $\varphi_1=\pi/3$; $r_0=1.0$; $Pe=4.5$ for different values of $a^*$:
*1* — $a^*=0$; *2* — 0.5; *3* — 1.0; *4* — 1.25; *5* — 1.5

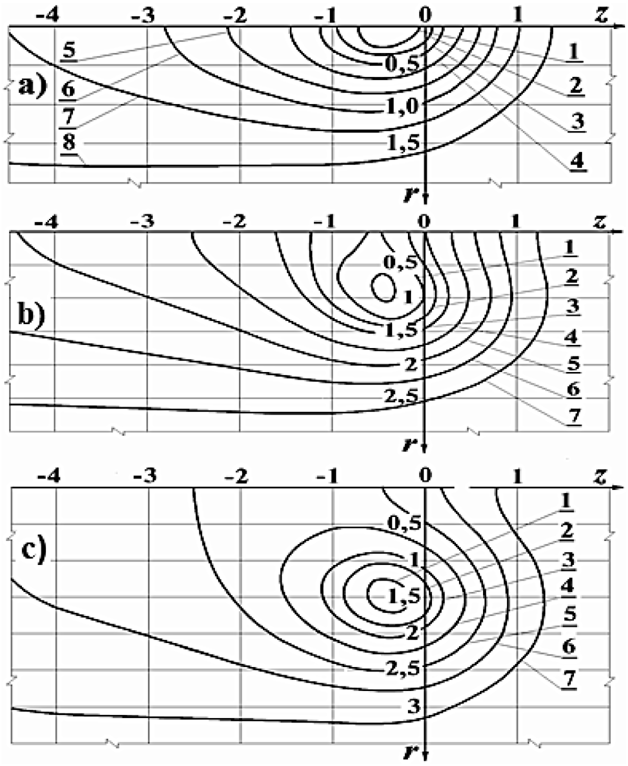


**Fig. 3.** Temperature field of the surface being treated of the cutting tool $\varphi = 0$ at $\varphi_1 = \pi/3$; $r_0 = 1.0$; $Pe = 4.5$ for different values of $a^*$:
*a*) $a^* = 0$; *1* — $\theta = 0.933$; *2* — 0.866; *3* — 0.8; *4* — 0.666; *5* — 0.533; *6* — 0.4; *7* — 0.266; *8* — 0.133;
*b*) $a^* = 1.0$; *1* — $\theta = 0.8$; *2* — 0.733; *3* — 0.666; *4* — 0.533; *5* — 0.4; *6* — 0.266; *7* — 0.133;
*c*) $a^* = 1.5$; *1* — $\theta = 0.8$; *2* — 0.733; *3* — 0.666; *4* — 0.533; *5* — 0.4; *6* — 0.266; *7* — 0,133

on the surface being treated and goes beyond the cutting edge is practically not used to heat the tool. Moreover, the value of this unused heat flux is the greater, the closer to the cutting edge is the maximum of the heat supply diagram. That is, under the conditions under consideration, the total heat flux $Q$ supplied to the tool during quenching increases with increasing distance from the cutting edge to the location of the maximum heat input. It is also important to note the fact that this increase is most significant in the region of relatively small values of $a^*$. These circumstances should be borne in mind in the further analysis of the specific features of the influence of the parameter $a^*$ on the thermal state of the considered detail.

According to the results of computational experiments, the change in $a^*$ has a significant impact on the nature of the temperature fields in the tool being treated. The relevant data is illustrated in Fig. 2 and 3. As can be seen from fig. 2, the change in tool temperature along the radius is different for different values of $a^*$. With relatively small $a^*$ (lines *1*, *2* on the graph), the maximum temperature takes place on the cutting edge itself, and as it moves away from it, the temperature monotonously decreases. For sufficiently large $a^*$ (lines *3*, *4*, *5*), the dependence $\theta = f(r)$ is extreme. The maximum temperature value is shifted relative to the cutting edge, the more significant the larger the value of $a^*$.

Interpreting the data presented, it is necessary to pay attention to the following points. The peculiarities of the change in the maximum temperature in a fixed section $z$ = const depending on the distance $a^*$ are related to the influence of certain competing factors: first, the total heat flow $Q$ supplied to the part, and second, the conditions of heat flow to the detail. As already noted, as $a^*$ increases, the total heat flow to the tool increases. This, obviously, should lead to a tendency for the object temperature to increase. However, this improves the conditions for heat outflows from the heating spot, which leads to a decrease in the maximum temperature of the detail. As follows from the data obtained, in the region of small values of $a^*$ (lines *1* and *2* in Fig. 2), the action of the first of the noted factors dominates, at sufficiently large $a^*$ (lines *3*, *4*, *5*) — the second of factors dominates.

In addition to this, it is also of interest to pay attention to the fact that the maximum temperature in the section $z$ = const with very large $a^*$ changes insignificantly with increasing $a^*$ (lines *4* and *5* in Fig. 2). This circumstance is a direct consequence of the effect of the regional influence of the geometric factor. Namely, as the heating spot is removed from the cutting edge (i.e. with increasing $a^*$), the unheated surface of the tool $\varphi = \varphi_1$ has less and less influence on the temperature distribution in the high-temperature zone, so that starting from a certain value $a^*$ the maximum in this section $z$ = const, the temperature $\theta_{max}$ becomes almost unchanged. General ideas about the nature of the location of the isotherms on the heated surface of the tool for various $a^*$ give Fig 3 *a*), *b*) and *c*). According to the maps of isotherms, there is a significant difference in their configurations for small and large values of $a^*$, which, obviously, is due to the influence of the factors noted above.

Figure 4 illustrates the dependence on the size $a^*$ of the dimensions of the hardened zone $b$ and $c$, respectively, on the surface being treated $\varphi = 0$ and unworked $\varphi = \varphi_1$ tool surfaces. In the figure, the lines with crosses correspond to the obtained numerical solutions, and the lines with circles — to experiments. As can be seen, the value of $b$ increases significantly with a raising in the distance $a^*$, reaches a maximum and then decreases sharply enough. The indicated increase in $b$ is obviously associated with a raising in the total heat flux $Q$ supplied to the detail, and its decrease is due to the improvement of heat outflows from the heating spot to the detail massif. As for the size of the hardened zone $c$ on the surface $\varphi = \varphi_1$, then, firstly, it turns out to be significantly less than the value of $b$. And secondly, when $a^* > 1.3$, the value of $c$ becomes equal to zero, i.e. the hardened zone does not reach the unworked tool surface $\varphi = \varphi_1$.

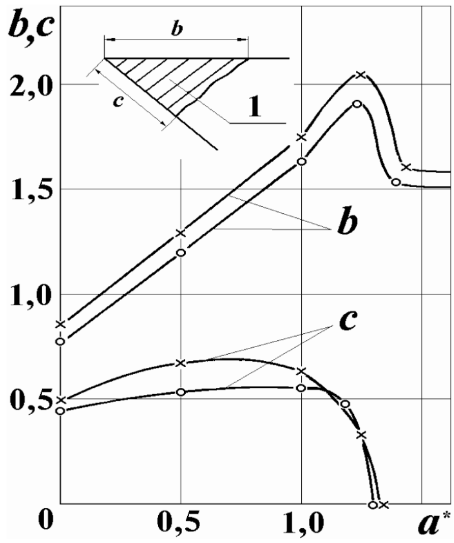


**Fig. 4.** The dependence of the sizes *b* and *c* the hardened zone *1* on the distance *a** between the cutting edge and the position of the center of the laser beam at $\varphi_1 = \pi/3$; $r_0 = 1.0$; $Pe = 4.5$;
—o— – experimental data; —×— – result of numerical solution

## Conclusions

1. The studies of thermal phenomena in the implementation of the technology of hardening the cutting tool with continuous laser radiation were performed.

2. The possibilities of influencing the thermal state of the hardened tool by varying the distance between the cutting edge and the center of the beam were studied.

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# МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССОВ ТЕПЛОПЕРЕНОСА ПРИ ЛАЗЕРНОЙ ЗАКАЛКЕ НА ОСНОВЕ МЕТОДОВ ПОЛИАРГУМЕНТНЫХ СИСТЕМ

***Представлены результаты компьютерного моделирования трехмерных температурных полей режущего инструмента в условиях его упрочнения непрерывным излучением лазера. Приводятся основные положения построения метода полиаргументных систем, используемого при решении рассматриваемой квазистационарной задачи теплопереноса. Анализируются результаты выпол-***

***ненных вычислительных экспериментов по изучению эффектов влияния на тепловое состояние режущего инструмента расстояния а* между режущей кромкой и центром лазерного луча. По данным проведенных исследований показано, что изменение величины а* оказывает существенное влияние на характер температурных полей в обрабатываемом инструменте. Представлены результаты численных исследований и натурных экспериментов, касающиеся зависимости размеров упрочненной зоны от расстояния а* между режущей кромкой и центром пятна нагрева. Приводится физическая интерпретация полученных данных на основе анализа действия ряда конкурирующих факторов.*** [dx.doi.org/10.29010/086.3]

*Ключевые слова: лазерная закалка; процессы теплопереноса; методы полиаргументных систем; математическое моделирование, режущий инструмент.*

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